Nonlinear Transformation Methods for Gravity-Turn Descent

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Introduction

GRAVITY-TURN guidance has been widely investigated[1–3] and used in practice[4,5] for terminal descent to a planetary surface. Previous studies of this method have utilized simple linear feedback laws to track predefined descent profiles. This Note points out that such approaches have much in common with modern nonlinear transformation methods. Furthermore, nonlinear methods can now be used for the design of descent guidance in a systematic and rigorous manner, enhancing the stability domain of the system. These matters have also been investigated for atmospheric entry by applying nonlinear methods to the Space Shuttle reference drag guidance law.[6] Finally, a systematic new guidance law is presented that requires only slant range, slant range rate, and local vertical information. This law dispenses with the explicit need for a radar altimeter for terminal descent. Such a guidance law may have benefits for low-cost lunar landing vehicles using simple onboard sensors.[7]

Gravity-Turn Descent

Gravity turn is a simple descent method whereby the vehicle thrust vector is aligned opposite to the vehicle velocity vector at all points along the descent trajectory. This may be easily implemented onboard by using the vehicle attitude control system to null body rates about the vehicle velocity vector. The method is also near optimal providing minimum-fuel descents.

For terminal descent maneuvers the vehicle planar translational dynamics may be modeled by a point mass moving over a flat surface with a uniform local gravitational acceleration g, viz.,

$$\dot{v} = -ng + g\cos\psi \tag{1a}$$

$$v\dot{\psi} = -g\sin\psi \tag{1b}$$

$$\dot{h} = -v\cos\psi \tag{1c}$$

$$\dot{s} = v\sin\psi \tag{1d}$$

where the state variables are illustrated in Fig. 1. In addition, the slant range R is defined as the distance along the velocity vector from the vehicle to the lunar surface. This distance and its derivative can be easily measured by a fixed Doppler radar strapped down to the lander $-z$ body axis. The only control is the vehicle thrust-to-weight ratio n that is modulated by the descent engine throttle.

Velocity-Altitude Tracking

A constant thrust-to-weight ratio can be found that will ensure a soft landing of the vehicle.[2] The resulting descent profile, however, is open loop and is sensitive to errors in descent engine ignition altitude and thrust magnitude. Therefore, to ensure a safe descent it is desirable to track a predefined velocity-altitude profile. This is possible with the vehicle thrust vector fixed opposite to the velocity vector in the gravity-turn attitude. Such profile tracking was used, for example, for the Viking descent to the Martian surface.[5] The desired descent contour was compared to the true vehicle velocity, yielding an error signal for the descent engine throttle. The required acceleration to track the predefined contour could be obtained in a manner somewhat similar to the contemporary feedback linearization method.[6] It will now be demonstrated that the true feedback linearization method may be used to generate explicit commands that will give exact error dynamics yielding robust tracking of the required descent contour.

It will be assumed that the desired altitude $\tilde{h}$ is defined by some function of the vehicle velocity v as

$$\tilde{h} = f(v) \tag{2}$$

Then a pseudocontrol u can be defined as

$$u \stackrel{\Delta}{=} h - f(v) \tag{3}$$

Derivatives of this pseudocontrol are then taken along the vehicle trajectory (i.e., Lie derivative) until the control n appears explicitly. This occurs with the first derivative, viz.,

$$\dot{u} = \dot{h} - f'(v)\dot{v} \tag{4}$$

where $f'(v)$ is the derivative of the function f with respect to the vehicle velocity. To ensure convergence to the desired descent contour, the following identity is made:

$$\dot{u} = -\kappa u \tag{5}$$

The constant κ is chosen to obtained a suitable damped response. Substituting for $\dot{v}$ and $\dot{h}$ from Eqs. (1) the required thrust-to-weight ratio $\tilde{n}$ can be obtained from Eq. (4) as

$$\tilde{n} = \cos\psi + \frac{1}{g}\left\{\frac{v\cos\psi - \kappa(h - \tilde{h})}{f'(v)}\right\} \tag{6}$$

This equation is, in fact, identical to Eq. (16) in Ref. 5 but with the addition of the altitude error term. This ensures that the error dynamics behave exactly as a damped first-order system. In principle, therefore, the controller ensures tracking of the desired descent contour from all initial points in the system state space, although actuator saturation and state constraints clearly limit this property.

Velocity-Slant Range Tracking

The procedure used in the preceding section can be utilized to design a guidance law to track a desired velocity-slant range profile. Using slant range rather than altitude as the independent variable eliminates the explicit need for radar altimeter data during the terminal descent. Slant range and range rate may be obtained from a Doppler radar strapped to the $-z$ body axis of the vehicle, Fig. 1.

Following previous studies,[1,3,4] a descent profile of the form

$$\tilde{v} = \sqrt{2(n^* - 1)R} \tag{7}$$

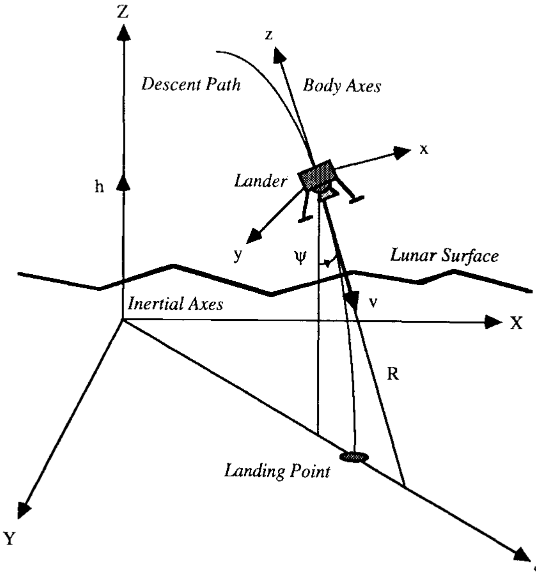

Fig. 1 Schematic geometry of a gravity-turn descent.

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will be used where n^* represents the nominal (constant) thrust-to-weight ratio required if the descent was vertical. Again, in previous studies this profile would be tracked by adding a simple linear feedback law with a high gain to guarantee tracking. The required thrust-to-weight ratio $\tilde{n}$ is then written as

$$\tilde{n} = n^* + \lambda(v - \tilde{v}) \tag{8}$$

This control will provide adequate tracking, however, only if the gain λ is large enough to compensate for the nonlinear terms in the vehicle dynamics represented by Eqs. (1). Using nonlinear transformation methods it will now be demonstrated that the desired descent profile can be tracked in a systematic manner using slant range, slant range rate, and local vertical information.

A pseudocontrol will again be defined as

$$u \stackrel{\Delta}{=} v - \tilde{v} \tag{9}$$

where $\tilde{v}$ is the required vehicle velocity defined as a function of the measured slant range. Taking the first derivative of the pseudocontrol, the thrust-to-weight ratio appears as before, viz.,

$$\dot{u} = \dot{v} - \tilde{v}'(R)\dot{R} \tag{10}$$

where, again, the slant range and range rate can be measured with a body-fixed Doppler radar strapped to the $-z$ body axis of the descent vehicle. The required thrust-to-weight ratio $\tilde{n}$ can then be obtained as

$$\tilde{n} = \cos\psi + (1/g)\{-\tilde{v}'(R)\dot{R} + \kappa(v - \tilde{v})\} \tag{11}$$

To implement this control requires measurements of slant range and range rate, vehicle velocity, and the descent angle ψ. This corresponds to a knowledge of the local vertical. Such information can be derived from a horizon sensor or even from inertial navigation system estimates updated from the vehicle predescent attitude.[3] Importantly, the control ensures that the error dynamics behave exactly as a damped first-order system, allowing tracking of the desired decent profile from a large domain of the system state space.

A typical descent profile is shown in Fig. 2a for a starting condition well away from the desired descent contour. The contour is defined by Eq. (7) with a nominal thrust-to-weight ratio n^* of 2.35. The initial vehicle velocity is some 50% larger than the required velocity on the descent contour. It can be seen from Fig. 2b that the initial error is damped by throttling the descent engine so that the desired descent contour is accurately tracked. A 5-g axial load limiter has been included. Since the vehicle velocity is monotonically decreasing, however, the desired descent contour is eventually reached and subsequently tracked.

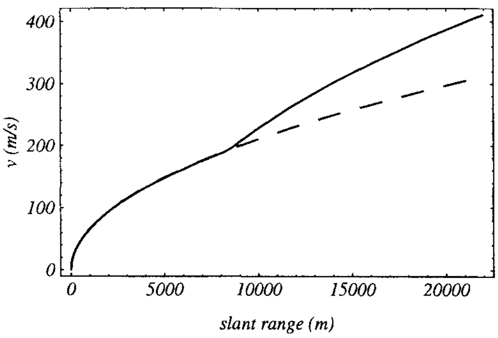

Fig. 2a Velocity-slant range profile tracking: – – –, desired profile, and ——, actual profile.

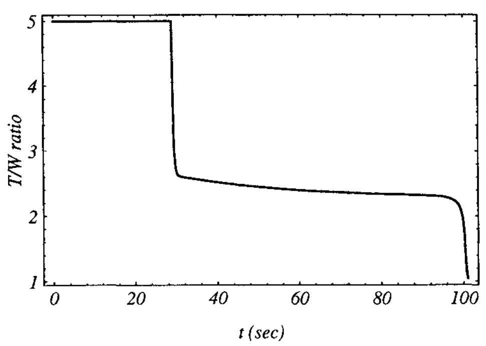

Fig. 2b Commanded thrust-to-weight ratio T/W.

Conclusions

It has been demonstrated that previous studies of descent contour tracking have much in common with contemporary nonlinear transformation methods. Rather than enforcing a high-gain, linear control over the desired decent contour, feedback linearization methods allow descent contour tracking in a systematic manner. The error dynamics of the systems are designed to exactly follow a damped first-order linear system, ensuring tracking from off the desired descent contour. Using this method a descent guidance law has been derived that requires only simple sensor measurements to implement. This scheme may allow simple guidance with less complex sensors for low-cost soft landing of payloads on the lunar surface.

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Analytical Solution for Dynamic Analysis of a Flexible L-Shaped Structure

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I. Introduction

DYNAMIC analysis of flexible structures usually starts from mathematical modeling of the structures. Both analytical approaches[1] and finite dimensional approximations[2] are used to derive the mathematical model. For simple structures, the analytical approach is an attractive choice in terms of the solution accuracy. In the rather general cases, however, the approximation techniques are frequently used due to computational advantage.

Analytical solutions rely upon solving boundary value problems represented by characteristic equations.[1,2] In spite of the difficulty of solving characteristic equations, the analytical approaches yield well-behaved shape functions as solutions for low-order variables, such as displacements and slopes, as well as higher order variables, such as strain and stress. On the other hand, approximation

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